

3.6

$$1 - (i) \quad \frac{1}{2x} + \frac{1}{3y} = 2 \quad \& \quad \frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$$

$$\frac{1}{x} = u \quad \& \quad \frac{1}{y} = v$$

$$\frac{u}{2} + \frac{v}{3} = 2 \Rightarrow 3u + 2v = 12 \quad \text{--- (1)}$$

Also,

$$\frac{u}{3} + \frac{v}{2} = \frac{13}{6} \Rightarrow 2u + 3v = 13 \quad \text{--- (2)}$$

$$3u + 2v = 12 \quad] \times 2$$

$$2u + 3v = 13 \quad] \times 3$$

$$~~6u + 4v = 24~~$$

$$~~6u + 9v = 39~~$$

$$-5v = -15$$

$$\Rightarrow \boxed{v = 3}$$

put in (1)

$$3u + 6 = 12$$

$$3u = 6$$

$$\boxed{u = 2}$$

$$\Rightarrow u = 2 \Rightarrow \frac{1}{x} = 2 \Rightarrow \boxed{x = \frac{1}{2}}$$

$$\Rightarrow v = 3 \Rightarrow \frac{1}{y} = 3 \Rightarrow \boxed{y = \frac{1}{3}}$$

$$\boxed{x = \frac{1}{2}}$$

$$\boxed{y = \frac{1}{3}}$$

$$\text{ii) } \frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2 \quad \& \quad \frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$

put $\frac{1}{\sqrt{x}} = u$ & $\frac{1}{\sqrt{y}} = v$

$$2u + 3v = 2 \quad \text{--- (1)}$$

$$4u - 9v = -1 \quad \text{--- (2)}$$

$$2u + 3v = 2 \quad] \times 2$$

$$\begin{array}{r} 4u - 9v = -1 \\ 4u + 6v = 4 \\ \hline -15v = -5 \end{array}$$

$$\boxed{v = \frac{1}{3}}$$

put in (1)

$$2u + 1 = 2$$

$$\Rightarrow \boxed{u = \frac{1}{2}}$$

$$\Rightarrow u = \frac{1}{2} \Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow \boxed{x = 4}$$

$$\Rightarrow v = \frac{1}{3} \Rightarrow \frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow \boxed{y = 9}$$

$$(i) \frac{5}{x-7} + \frac{1}{y-2} = 2$$

$$\frac{6}{x-1} - \frac{3}{y-2} = 1$$

put $\frac{1}{x-1} = u$ & $\frac{1}{y-2} = v$

$$5u + v = 2 \quad] \times 3 \quad \text{--- (1)}$$

$$6u - 3v = 1 \quad \text{--- (2)}$$

$$15u + 3v = 6$$

$$6u - 3v = 1$$

$$21u = 7$$

$$\boxed{u = \frac{1}{3}}$$

put in (1)

$$\frac{5}{3} + v = 2$$

$$\Rightarrow \boxed{v = \frac{1}{3}}$$

$$\Rightarrow u = \frac{1}{3} \Rightarrow \frac{1}{x-1} = \frac{1}{3} \Rightarrow \boxed{x=4}$$

$$\text{Also, } v = \frac{1}{3} \Rightarrow \frac{1}{y-2} = \frac{1}{3} \Rightarrow \boxed{y=5}$$

$$(v) \quad \frac{7x-2y}{xy} = 5 \quad \& \quad \frac{8x+7y}{xy} = 15$$

$$\frac{7x}{xy} - \frac{2y}{xy} = 5 \quad \& \quad \frac{8x}{xy} + \frac{7y}{xy} = 15$$

$$\frac{7}{y} - \frac{2}{x} = 5 \quad \& \quad \frac{8}{y} + \frac{7}{x} = 15$$

$$\rightarrow \text{put } \frac{1}{x} = u \quad \& \quad \frac{1}{y} = v$$

$$= \quad \left. \begin{array}{l} 7v - 2u = 5 \end{array} \right] \times 8 \quad \text{--- (1)}$$

$$\left. \begin{array}{l} 8v + 7u = 15 \end{array} \right] \times 7 \quad \text{--- (2)}$$

~~$$56v - 16u = 40$$~~

~~$$56v + 49u = 105$$~~

$$-65u = -65$$

$$\Rightarrow \boxed{u = 1}$$

put in (1)

$$7v - 2 = 5 \quad \Rightarrow \quad \boxed{v = 1}$$

$$\Rightarrow u = 1 \quad \Rightarrow \quad \frac{1}{x} = 1 \quad \Rightarrow \quad \boxed{x = 1}$$

$$\Rightarrow v = 1 \quad \Rightarrow \quad \frac{1}{y} = 1 \quad \Rightarrow \quad \boxed{y = 1}$$

$$(viii) \frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4} \quad \& \quad \frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = -\frac{1}{8}$$

$$\text{put } \frac{1}{3x+y} = u \quad \& \quad \frac{1}{3x-y} = v$$

$$u + v = \frac{3}{4} \Rightarrow 4u + 4v = 3 \quad \text{--- (1)}$$

$$\frac{u}{2} - \frac{v}{2} = -\frac{1}{8} \Rightarrow 4u - 4v = -1 \quad \text{--- (2)}$$

$$\begin{array}{r} 4u + 4v = 3 \\ 4u - 4v = -1 \\ \hline \end{array}$$

$$8u = 2$$
$$\boxed{u = \frac{1}{4}}$$

put in (1)

$$1 + 4v = 3$$
$$\boxed{v = \frac{1}{2}}$$

$$u = \frac{1}{4} \Rightarrow \frac{1}{3x+y} = \frac{1}{4} \Rightarrow 3x+y = 4 \quad \text{--- (3)}$$

$$v = \frac{1}{2} \Rightarrow \frac{1}{3x-y} = \frac{1}{2} \Rightarrow 3x-y = 2 \quad \text{--- (4)}$$

$$\begin{array}{r} 3x + y = 4 \\ 3x - y = 2 \\ \hline 6x = 6 \\ \boxed{x = 1} \end{array}$$

put in (3)

$$3 + y = 4$$
$$\boxed{y = 1}$$

Let x be the speed of rowing in still water = x km/hr
Let speed of current = y km/hr

Downstream (along the current):-

Total speed = $(x+y)$ km/hr

Distance = 20 km

Time = 2 hrs.

$$\Rightarrow x+y = \frac{20}{2}$$

$$x+y = 10 \quad \text{--- (1)}$$

Upstream (against the current):-

Total speed = $(x-y)$ km/hr

Distance = 4 km

Time = 2 hr

$$\Rightarrow x-y = \frac{4}{2}$$

$$\Rightarrow x-y = 2 \quad \text{--- (2)}$$

$$x+y = 10$$

$$2x = 12$$

$$\boxed{x = 6}$$

put in (1)

$$6-y = 2$$

$$\boxed{y = 4}$$

$$v = \frac{d}{t}$$

\Rightarrow speed of rowing in still water = 6 km/hr
speed of current = 4 km/hr