

VECTOR

Dot Product

$$A = A_1 \hat{i} + A_2 \hat{j} + A_3 \hat{k}$$

$$B = B_1 \hat{i} + B_2 \hat{j} + B_3 \hat{k}$$

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

Example

$$\vec{A} = 3\hat{i} + 5\hat{j} + 10\hat{k}$$

$$A = \sqrt{9 + 25 + 100} = \sqrt{134}$$

$$\vec{B} = 4\hat{i} + 5\hat{j} + 9\hat{k}$$

$$B = \sqrt{16 + 25 + 81} = \sqrt{122}$$

$$\vec{A} \cdot \vec{B} = 12 + 25 + 90$$

$$= 127$$

$$\cos \theta = \frac{127}{\sqrt{134} \times \sqrt{122}}$$

Note

$$\vec{A} \perp \vec{B} \rightarrow \theta = 90$$

$$\vec{A} \cdot \vec{B} = 0$$

Cross Product

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

Example

$$\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{B} = 2\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 2 \end{vmatrix} = (4 - 6)\hat{i} - (2 - 6)\hat{j} + (3 - 4)\hat{k}$$

$$= -2\hat{i} + 4\hat{j} - 1\hat{k}$$

Direction of vector \vec{A} [unit vector of \vec{A}]

$$\hat{a} = \frac{\vec{A}}{|\vec{A}|} = \frac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{14}}$$

Projection

