

## 'PROJECTILE MOTION'

If velocity of particle in one direction is const & Acceleration which is in  $\perp$  direction remains same w.r.t Motion of particle is called projectile & its path is always parabolic.

### Ground - On ground projection

$$T = \frac{2u \sin \theta}{g} = \frac{2u y}{g} \leftarrow \text{time of flight.}$$

$$h_{\max} = \frac{u^2 \sin^2 \theta}{2g} = \frac{2y}{2g} \rightarrow \text{max height.}$$

$$R = (u \cos \theta) T = \frac{2u x u y}{g} = \frac{u^2 \sin \theta}{g}$$

$$\vec{v} = (u \cos \theta) \hat{i} + (u \sin \theta - gt) \hat{j}$$

$$\alpha = (u \cos \theta t) \hat{i} + (u \sin \theta t - \frac{1}{2} g t^2) \hat{j}$$

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta} = x \tan \theta \left[ 1 - \frac{x}{R} \right]$$

\* Velocity at time 't'

Horizontal

$$v_x = u \cos \theta \approx \theta^\circ$$

Vertical

$$v_y = (u \sin \theta) - gt$$

\* Angle of velo. from ~~x~~-axis  
 $\alpha = \tan^{-1} \left( \frac{v_y}{v_x} \right) = \tan^{-1} \left( \frac{u \sin \theta - gt}{u \cos \theta} \right)$

\* Disp. at time 't'

$$|\vec{r}| = \sqrt{x^2 + y^2}$$

- \* From bottom to top  $\alpha \uparrow$
- \* At top position  $\alpha = 0$
- \* From top to bottom  $\alpha \uparrow$

\* Horizontal Range.

$$R = \frac{u^2 \sin 2\theta}{g}$$

\* K.E at lowest point.

$$K.E. = \frac{1}{2} m u^2$$

NOTE → \* Horizontal component of velocity remains same all points of its path  
\* At top point path of particle is circular & resultant velo. remains same.

### Special case for on ground to on ground projection

#### case-I → Max range condition

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$* u = c, g = c \Rightarrow R = f(\theta)$$

$$\downarrow_{\max} (\sin 2\theta) = 1$$

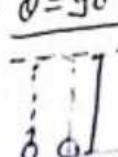
$$\downarrow_{2\theta} = 90^\circ, \pi/2$$

$$\theta = 45^\circ / \pi/4$$

$$\# R_{\max} = \frac{u^2}{g} = R_{45^\circ}$$

$$h_{45^\circ} = \frac{u^2 \sin^2 45^\circ}{2g} = \frac{u^2}{4g}$$

$$\# \theta = 90^\circ \text{ (vertical projection)}$$



$$R_{90^\circ} = 0$$

$$h_{90^\circ} = \frac{u^2}{2g} = \frac{R_{45^\circ}}{2} \Rightarrow R_{45^\circ} = 2h_{90^\circ}$$

$$T_{\max} = \frac{2u}{g} = T_{90^\circ}$$

$$h_{45^\circ} = \frac{R_{45^\circ}}{4} = \frac{h_{90^\circ}}{2}$$

$$\begin{aligned} R_{\max} \Rightarrow \theta &\Rightarrow 45^\circ \\ H_{\max} \Rightarrow \theta &\Rightarrow 90^\circ \\ T_{\max} \end{aligned}$$

1a) $\theta \leq 0 \leq 45^\circ$	$\theta \uparrow, T \uparrow, h_{\max} \uparrow, R \uparrow$
1b) $45^\circ < \theta < 90^\circ$	$\theta \uparrow, T \uparrow, h_{\max} \uparrow, R \uparrow$